FYUGP/1st Sem/25(NEP)New

2025

Four Year Under Graduate Programme (FYUGP)

1st Semester Examination (Under NEP)
(New Session 2024-25)

MATHEMATICS (Major)

Paper Code : MTMMJ DC-101

[Algebra]

Full Marks : 50 Time : Two Hours

The figures in the margin indicate full marks. Candidates are required to give their answers in their own words as far as practicable.

Notations and symbols have their usual meanings.

Answer question no. 1 and any *four* from the rest.

1. Answer any *five* questions : $2\times5=10$

(a) State the Fundamental Theorem of Arithmetic.

(b) Give an example of a relation on $\mathbb{Z}$ which is reflexive, symmetric but not transitive.

(c) Find the general and principal value of i^i.

(d) Show that the greatest integer function $f:\mathbb{R}\to\mathbb{R}$ gvien by $f(x)=[x], x\in\mathbb{R}$, is neither one-one nor onto.

(e) Prove that $(a+b+c)\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)\geq 9$, where a, b, c are positive real numbers.

(f) Show that the product of all the values of $\left(1+i\sqrt{3}\right)^{\frac{3}{4}}$ is 8.

(g) Find the number of non-real roots of $x^4+2x^2+3x-4=0$.

(h) Find the remainder when 1! + 2! + 3! + ... + 50! is divided by 15.

2. (a) Applying De Moivre's theorem, prove that

$$\sin^4\theta\cos^2\theta=\frac{1}{32}[\cos 6\theta-2\cos 4\theta-\cos 2\theta+2].$$

(b) Prove that

$$\left(\frac{1+\sin\phi+i\cos\phi}{1+\sin\phi-i\cos\phi}\right)^n=\cos\left(\frac{n\pi}{2}-n\phi\right)+i\sin\left(\frac{n\pi}{2}-n\phi\right)$$

(c) If a, b and c are three unequal positive real numbers, then prove that

$$\frac{2}{b+c}+\frac{2}{c+a}+\frac{2}{a+b}>\frac{9}{a+b+c}.$$

4+3+3

3. (a) Solve the following biquadratic equation by Ferrari's method $2x^4+6x^3-3x^2+2=0$.

(b) If $a_1, a_2, \ldots, a_n$ be n positive rational numbers such that $s = a_1 + a_2 + \ldots + a_n$, then prove that

$$\left(\frac{s}{a_1}-1\right)^{a_1}\left(\frac{s}{a_2}-1\right)^{a_2}\cdots\left(\frac{s}{a_n}-1\right)^{a_n} \leq (n-1)^s.$$

(c) If α, β, γ be the roots of the equation $x^3-3x+1=0$, then find the equation whose roots are $\beta^2+\gamma^2-\alpha^2,\ \gamma^2+\alpha^2-\beta^2, \alpha^2+\beta^2-\gamma^2$.

5+2+3

4. (a) If the roots of the equation $x^3-px^2+qx-r=0$ are α, β, γ, find the value of $(\beta+\gamma)(\gamma+\alpha)(\alpha+\beta)$.

(b) If α be a multiple root of order 3 of the equation $x^4+bx^2+cx+d=0, (d \neq 0)$, show that

$\alpha = -\dfrac{8d}{3c}$.

(e) Find the greatest value of $(2x+1)^3(y+2)^2$ when $x + y = 3$ and $-\frac{1}{2} < x < 5$.

2+4+4

P.T.O.

5. (a) Consider the recurrence relation :

$$a_n = 4a_{n-1} - 4a_{n-2} \cdot$$

Find the general solution of the recurrence relation. Also, find the solution when $a_0 = 1$ and $a_1 = 2$.

(b) Use the theory of congruences to prove that for all $n \geq 1, 7 \mid 2^{5n+3} + 5^{2n+3}$.

(c) Examine it the relation R in the set $\mathbb{Z}$ is an equivalence relation, where

$R = \{(a,b) \in \mathbb{Z} \times \mathbb{Z} : 3a + 4b$ is divisible by 7?

6. (a) Solve the system of linear equations

$$\begin{aligned} x + 2y + z - 3w &= 1 \\ 2x + 4y + 3z + w &= 3 \\ 3x + 6y + 4z - 2w &= 4 \end{aligned}$$

(b) Let A be a 3×3 matrix whose eigenvalues are 1, −1, 0. Find det $(A^{100} + I_3)$.

(c) Show that the matrix A and A^T have same eigenvalues.

7. (a) Verify Cayley-Hamilton Theorem for the matrix

$$A = \begin{pmatrix} 1 & 2 & 1 \\ 1 & -1 & 1 \\ 2 & 3 & -1 \end{pmatrix}.$$ Hence find A^{-1}.

(b) Determine the nature of the quadratic form $x^2+2y^2+3z^2-2xy+4yz$. Also determine its rank and signature. (3+2)+(4+1)
